Int. J. Heat Mass Transfer. Vol. 35, No. 3, pp. 613–621, 1992
Printed in Great Britain

0017–9310/92 $5.00 + 0.00
Pergamon Press plc

Onset of natural convection under an electric field†

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Abstract—The convective instability problem in a.c. and d.c. electric fields is studied theoretically. Linearized perturbation equations are solved by the Galerkin method where the critical a.c. or d.c. electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number are expressed as eigenvalues. Nondimensional parameters, Biot number *Bi*, a.c. electric Rayleigh number *Ea*, Rayleigh number *Ra* and Marangoni number *Ma*, determine the onset of natural convection in the case of the a.c. electric field, while parameters *Bi*, *S*, *K*, *Ea*, *Ra* and *Ma* become the governing factors in the case of the d.c. electric field. It is found that the critical a.c. electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number become smaller as the electric field increases in the case of the a.c. electric field. However, in the case of the d.c. electric field, the critical d.c. electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number increase as the electric field increases when $|S|$ and K are small, while the critical d.c. electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number decrease as the electric field increases when $|S|$ and K are large.

INTRODUCTION

THE CONVECTIVE instability problem in a horizontal liquid layer has attracted many researchers' interest since Bénard's successful work [1]. The onset of buoyancy convection was studied in an excellent way by Chandrasekhar [2]. Natural convection driven by surface tension was first analysed theoretically by Pearson [3] and he showed that the Bénard cell was caused by the effect of surface tension. Convection driven by both buoyancy and surface tension was studied by Nield [4].

Onset of natural convection in an external field such as a magnetic field or an electric field has also been studied by several researchers. Chandrasekhar [2] analysed the onset of natural convection in a magnetic field. The effect of the magnetic field on the onset of convection driven by both buoyancy and surface tension was also studied by Nield [5]. The convective instability problem in a.c. or d.c. electric fields has been studied by Roberts [6], Turnbull [7–9], Turnbull and Melcher [10] and Takashima and Aldridge [11].

We have studied the effect of the magnetic field on the flow and temperature characteristics of natural convection and clarified the conditions under which natural convection is suppressed [12–15]. As a next step, we have started studying the effect of the electric field on natural convection, selecting it as another external field. In a liquid layer under an electric field, the electric force is induced by the nonuniformity of both electric conductivity and dielectric constant, which in general depend on both the concentration and temperature [16].

In this paper, the onset of electric convection, buoyancy convection and Marangoni convection in a.c. and d.c. electric fields is analysed theoretically and the effect of the electric field on the critical a.c. and d.c. electric Rayleigh numbers, the critical Rayleigh number and the critical Marangoni number is clarified.

GOVERNING EQUATIONS

The electric force which acts on fluid per unit volume is expressed as follows [16]:

$$\mathbf{f}_e = \text{grad}\left(\tfrac{1}{2}\mathbf{E}^2\rho\frac{\partial\varepsilon}{\partial\rho}\right)+\tfrac{1}{2}\mathbf{E}^2\,\text{grad}\,\varepsilon+\rho_e\mathbf{E} \qquad (1)$$

where the first two terms represent forces induced by

† Dedicated to Professor Dr.-Ing. Dr.-Ing.e.h. Ulrich Grigull.

NOMENCLATURE

Bi	Biot number
$\mathbf{E}$	electric field
Ea	a.c. electric Rayleigh number, equation (11)
Ed	d.c. electric Rayleigh number, equation (28)
$\mathbf{f}_e$	electric force per unit volume
k	nondimensional wave number
$\mathbf{k}$	unit vector in the Z-direction
K	nondimensional parameter defined by equation (27)
L	depth of liquid layer
Ma	Marangoni number
p	pressure
s_1, s_2	temperature coefficients of conductivity, equation (22)
S	nondimensional parameter defined by equation (26)
t	time
T	temperature
$\mathbf{v}$	velocity
V_Z	Z-component of nondimensional velocity.

Greek symbols

α	temperature coefficient of dielectric constant, equation (7)
ΔT	temperature difference between top free surface and bottom rigid wall
ε	dielectric constant
θ	nondimensional temperature
κ	thermal diffusivity
ν	kinematic viscosity
ρ	density
ρ_e	free charge density
σ	electric conductivity
ϕ	electric potential
Φ	nondimensional electric potential.

Subscripts

c	critical value
0	initial value.

the nonuniformity of the dielectric constant and the last term is the Coulomb force.

The governing equations of electrohydrodynamics are expressed as follows:

(i) Continuity equation

$$\operatorname{div} \mathbf{v} = 0 \tag{2}$$

(ii) Momentum equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \operatorname{grad})\mathbf{v} = -\frac{1}{\rho}\operatorname{grad} p + \nu\Delta\mathbf{v} + \beta g(T-T_0)\mathbf{k} + \frac{1}{\rho}\mathbf{f}_e \tag{3}$$

(iii) Energy equation

$$\frac{\partial T}{\partial t} + (\mathbf{v} \operatorname{grad})T = \kappa\Delta T \tag{4}$$

(iv) Equations of electric field

$$\operatorname{rot} \mathbf{E} = 0, \quad \frac{\partial \rho_e}{\partial t} + \operatorname{div} \mathbf{J} = 0$$

$$\rho_e = \operatorname{div}(\varepsilon\mathbf{E}), \quad \mathbf{J} = \sigma\mathbf{E} + \rho_e\mathbf{v}. \tag{5}$$

The relation between electric potential and electric field is

$$\mathbf{E} = -\operatorname{grad} \phi. \tag{6}$$

ANALYSIS

The convective instability problem in a horizontal liquid layer, the top surface and the bottom of which are respectively free and rigid, is studied (see Fig. 1).

A.c. electric field

When an a.c. electric field, the frequency of which is 50 or 60 Hz, is applied, the Coulomb force is negligible because the electric relaxation time of most fluids is of the order of 10–100 s. Therefore, the force induced only by the nonuniformity of the dielectric constant is considered and the dielectric constant is assumed to depend linearly on the temperature in the present analysis

$$\varepsilon = \varepsilon_0[1-\alpha(T-T_0)]. \tag{7}$$

Nondimensional perturbation equations are obtained from the governing equations, neglecting the

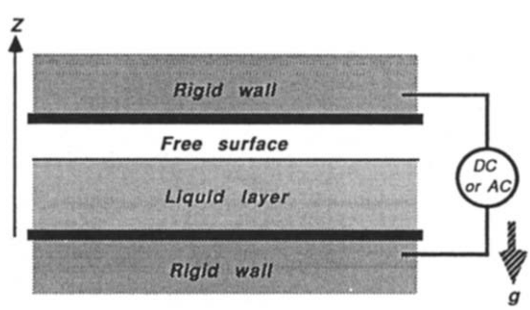

FIG. 1. Horizontal liquid layer under an electric field.

second order terms and assuming that time-dependent instability does not occur

$$\Delta^2 V_Z + Ra\,\Delta_{II}\theta + Ea\left[\Delta_{II}\theta + \frac{\partial}{\partial Z}\Delta_{II}\Phi\right] = 0 \quad (8)$$

$$\Delta\Phi + \frac{\partial\theta}{\partial Z} = 0 \quad (9)$$

$$V_Z + \Delta\theta = 0 \quad (10)$$

where Δ_{II} is a two-dimensional horizontal Laplacian and the coordinates, velocity, temperature and electric potential are nondimensionalized by L, κ/L, ΔT and $\alpha E_0 \Delta T L$, respectively. Ra is the Rayleigh number and Ea, defined by equation (11), is a nondimensional parameter which we call the a.c. electric Rayleigh number and which represents the ratio of electric force to viscous force:

$$Ea = \frac{\varepsilon_0 E_0^2 \alpha^2 \Delta T^2 L^2}{\rho_0 \kappa \nu}. \quad (11)$$

Harmonic analysis can be applied to analyse the perturbation equations (8)–(10)

$$[V_Z \theta \Phi]^{\mathrm{T}} = [F(Z)G(Z)E(Z)]^{\mathrm{T}} \exp[i(k_X X + k_Y Y)] \quad (12)$$

where k_X and k_Y are respectively the X and Y components of the nondimensional wave number and $k^2 = k_X^2 + k_Y^2$.

The perturbation equations for the a.c. electric field can be expressed as follows by substituting equation (12) into equations (8)–(10):

$$(D^2-k^2)^2 F(Z) - Ra\,k^2 G(Z) - Ea\,k^2[G(Z)+DE(Z)] = 0 \quad (13)$$

$$(D^2-k^2)E(Z) + DG(Z) = 0 \quad (14)$$

$$F(Z) + (D^2-k^2)G(Z) = 0 \quad (15)$$

where $D^n = \mathrm{d}^n/\mathrm{d}Z^n$.

The boundary conditions are expressed as below where two types of conditions for electric potential at the free surface, that is, $\Phi = 0$ and $\mathrm{d}\Phi/\mathrm{d}Z = 0$, are considered following Turnbull's analysis [7]

$$F(0) = DF(0) = G(0) = E(0) = 0$$

$$F(1) = 0, \quad D^2F(1) = -k^2\,Ma\,G(1)$$

$$DG(1) = -Bi\,G(1)$$

$$E(1) = 0 \quad \text{or} \quad DE(1) = 0 \quad (16)$$

where Bi and Ma are the Biot number and the Marangoni number, respectively.

$F(Z)$, $G(Z)$ and $E(Z)$ are expanded in a series of trial functions f_i, g_i and e_i, respectively

$$F(Z) = \sum_i \alpha_i f_i, \quad G(Z) = \sum_i \beta_i g_i, \quad E(Z) = \sum_i \gamma_i e_i \quad (17)$$

where α_i, β_i and γ_i are coefficients. In this analysis, the following trial functions are used:

$$f_i = (1-Z)Z^{i+1}$$

$$g_i = Z^i \text{ (for } Bi \neq \infty) \quad \text{or} \quad (1-Z)Z^i \text{ (for } Bi = \infty)$$

$$e_i = (1-Z)Z^i \quad \text{or} \quad Z^i. \quad (18)$$

Applying the Galerkin method to equations (13)–(15), the conditions for the onset of electric convection, buoyancy convection and Marangoni convection are obtained as follows.

Onset of electric convection

$$\det\left[\frac{1}{Ea}I - k^2 A_{11}^{-1}(A_{12} + A_{13}A_{33}^{-1}A_{32})A_{22}^{-1}A_{21}\right] = 0. \quad (19)$$

Onset of buoyancy convection

$$\det\left[\frac{1}{Ra}I - k^2\{A_{11} - Eak^2(A_{12} + A_{13}A_{33}^{-1}A_{32}) \times A_{22}^{-1}A_{21}\}^{-1}A_{12}A_{22}^{-1}A_{21}\right] = 0. \quad (20)$$

Onset of Marangoni convection

$$\det\left[\frac{1}{Ma}I + k^2\{A_{11} - k^2[(Ea+Ra)A_{12} - Ea\,A_{13}A_{33}^{-1}A_{32}]A_{22}^{-1}A_{21}\}^{-1}B_{12}A_{22}^{-1}A_{21}\right] = 0, \quad (21)$$

where

$$A_{11} = \int_0^1 D^2 f_m D^2 f_i\,\mathrm{d}Z + 2k^2\int_0^1 Df_m Df_i\,\mathrm{d}Z + k^4\int_0^1 f_m f_i\,\mathrm{d}Z$$

$$A_{12} = \int_0^1 f_m g_i\,\mathrm{d}Z, \quad A_{13} = \int_0^1 f_m De_i\,\mathrm{d}Z,$$

$$A_{21} = \int_0^1 g_m f_i\,\mathrm{d}Z$$

$$A_{22} = Bi\,g_m(1)g_i(1) + \int_0^1 Dg_m Dg_i\,\mathrm{d}Z + k^2\int_0^1 g_m g_i\,\mathrm{d}Z$$

$$A_{32} = \int_0^1 e_m Dg_i\,\mathrm{d}Z,$$

$$A_{33} = \int_0^1 De_m De_i\,\mathrm{d}Z + k^2\int_0^1 e_m e_i\,\mathrm{d}Z$$

$$B_{12} = Df_m(1)g_i(1).$$

Equations (19)–(21) are eigenvalue equations where $1/Ea$, $1/Ra$ and $1/Ma$ are eigenvalues and the a.c. electric Rayleigh number, Rayleigh number and Marangoni number corresponding to the maximum eigenvalues become the critical a.c. electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number, respectively.

D.c. electric field

When a d.c. electric field is applied, the Coulomb force becomes more dominant than the dielectric force [11]. Therefore, the dielectric force can be neglected in this case and the electric conductivity is assumed to be a quadratic function of temperature [8, 11]

$$\sigma = \sigma_0[1+s_1(T-T_0)+s_2(T-T_0)^2]. \quad (22)$$

The effect of the second-order term on instability will be discussed later.

Nondimensional perturbation equations are expressed as follows:

$$\Delta^2 V_Z + Ra\,\Delta_{\mathrm{II}}\theta - Ed\,\Delta_{\mathrm{II}}\Delta\Phi = 0 \quad (23)$$

$$\Delta\Phi - [1+2S(1-Z)]\frac{\partial\theta}{\partial Z} + 2S\theta + 2SKV_Z = 0 \quad (24)$$

$$V_Z + \Delta\theta = 0 \quad (25)$$

where electric potential is nondimensionalized by $s_1 E_0 \Delta T L$. Nondimensional parameters S, K and Ed, the latter being called the d.c. electric Rayleigh number, are defined as follows:

$$S = \frac{s_2 \Delta T}{s_1} \quad (26)$$

$$K = \frac{\varepsilon_0 \kappa}{\sigma_0 L^2} \quad (27)$$

$$Ed = \frac{\varepsilon_0 E_0^2 s_1 \Delta T L^2}{\rho_0 \kappa \nu}. \quad (28)$$

Harmonic analysis can be applied to the perturbation equations (23)–(25)

$$[V_Z \theta]^{\mathrm{T}} = [F(Z) G(Z)]^{\mathrm{T}} \exp[i(k_X X + k_Y Y)]. \quad (29)$$

The perturbation equations for the d.c. electric field can be expressed as follows substituting equation (29) into equations (23)–(25):

$$(D^2-k^2)^2 F(Z) - Ra\,k^2 G(Z) + Ed\,k^2[DG(Z) + 2S(1-Z)DG(Z) - 2SG(Z) - 2SKF(Z)] = 0 \quad (30)$$

$$F(Z) + (D^2-k^2)G(Z) = 0. \quad (31)$$

The boundary conditions are

$$F(0) = DF(0) = G(0) = 0, \quad F(1) = 0$$
$$D^2F(1) = -Ma\,k^2 G(1)$$
$$DG(1) = -Bi\,G(1). \quad (32)$$

Following a procedure similar to the case of the a.c. electric field, $F(Z)$ and $G(Z)$ are expanded in a series of trial functions f_i and g_i, respectively.

$$F(Z) = \sum_i \alpha_i f_i, \quad G(Z) = \sum_i \beta_i g_i. \quad (33)$$

The following trial functions are used:

$$f_i = (1-Z)Z^{i+1}$$
$$g_i = Z^i \,(\text{for } Bi \neq \infty) \text{ or } (1-Z)Z^i \,(\text{for Bi} = \infty). \quad (34)$$

Applying the Galerkin method to equations (30) and (31), the conditions for the onset of electric convection, buoyancy convection and Marangoni convection are obtained as follows.

Onset of electric convection

$$\det\left[\frac{1}{Ea}I - k^2 A_{11}^{-1}\{2SKB_{11} + (C_{12}+2SD_{12}-2SA_{12})A_{22}^{-1}A_{21}\}\right] = 0. \quad (35)$$

Onset of buoyancy convection

$$\det\left[\frac{1}{Ra}I + k^2\{A_{11} - 2k^2\,Ed\,SKB_{11} - k^2\,Ed(C_{12}+2SD_{12}-2SA_{12}) \times A_{22}^{-1}A_{21}\}^{-1}A_{12}A_{22}^{-1}A_{21}\right] = 0. \quad (36)$$

Onset of Marangoni convection

$$\det\left[\frac{1}{Ma}I - k^2\{A_{11} - 2k^2\,Ed\,SKB_{11} + k^2[Ra\,A_{12} - Ed(C_{12}+2SD_{12}-2SA_{12})] \times A_{22}^{-1}A_{21}\}^{-1}B_{12}A_{22}^{-1}A_{21}\right] = 0. \quad (37)$$

Table 1. Critical a.c. electric Rayleigh number and critical wave number

Bi	$\Phi = 0$	$\mathrm{d}\Phi/\mathrm{d}Z = 0$
0.0	$Ea_c = 758.9053$ $k_c = 2.116$	$Ea_c = 698.0715$ $k_c = 1.886$
0.1	775.2779 2.148	724.7655 1.935
1.0	885.9332 2.330	913.533 2.248
10.0	1182.668 2.647	1484.788 2.944
100.0	1325.410 2.738	1793.26 3.205
1000.0	1345.667 2.749	1838.939 3.238
10 000.0	1347.779 2.750	1843.730 3.242
∞	1348.015 2.750	1844.265 3.242

Table 2. Critical Rayleigh number and critical wave number under an a.c. electric field

	Bi						
Ea	0.0	0.1	1.0	10.0	100.0	1000.0	∞
(a) $\Phi = 0$ at free surface							
0.0	$Ra_c = 668.9983$	682.3602	770.5697	989.4917	1085.898	1099.124	1100.650
	$k_c = 2.086$	2.116	2.293	2.589	2.672	2.681	2.682
1.0	668.1170	681.4802	769.7004	988.6561	1085.080	1098.308	1099.834
	2.086	2.116	2.293	2.589	2.672	2.681	2.682
10.0	660.1850	673.5610	761.8762	981.135	1077.716	1090.967	1092.495
	2.086	2.117	2.293	2.589	2.672	2.682	2.683
20.0	651.3718	664.7618	753.1827	972.7779	1069.533	1082.809	1084.341
	2.086	2.117	2.294	2.590	2.673	2.682	2.683
50.0	634.9315	638.3637	727.1014	947.7060	1044.985	1058.337	1059.877
	2.088	2.118	2.295	2.591	2.674	2.684	2.685
100.0	580.8635	594.3658	683.6306	905.9162	1004.069	1017.545	1019.100
	2.090	2.120	2.297	2.594	2.677	2.686	2.687
200.0	492.7232	506.3655	569.6817	822.3242	922.2235	935.9507	937.5345
	2.094	2.124	2.301	2.598	2.681	2.691	2.692
500.0	228.2693	242.3281	335.7752	571.4484	676.5912	691.0723	692.7436
	2.106	2.136	2.314	2.613	2.696	2.706	2.707
1000.0	−212.6013	−197.8593	−99.27122	152.9856	266.8759	282.6191	284.4369
	2.126	2.157	2.335	2.638	2.722	2.731	2.732
2000.0	−1094.797	−1078.729	−970.148	−685.235	−553.829	−535.542	−533.4289
	2.168	2.199	2.379	2.690	2.774	2.784	2.785
(b) $d\Phi/dZ = 0$ at free surface							
0.0	$Ra_c = 668.9983$	682.3602	770.5697	989.4917	1085.898	1099.124	1100.650
	$k_c = 2.086$	2.116	2.293	2.589	2.672	2.681	2.682
1.0	668.0478	681.4247	769.7256	988.8424	1085.328	1098.565	1100.092
	2.085	2.116	2.293	2.589	2.672	2.682	2.683
10.0	659.4927	673.0051	762.1289	982.9969	1080.201	1093.536	1095.074
	2.083	2.114	2.292	2.591	2.675	2.685	2.686
20.0	649.9850	663.6485	753.6882	976.4996	1074.499	1087.942	1089.493
	2.080	2.111	2.292	2.594	2.678	2.688	2.689
50.0	621.4498	635.5696	728.3670	956.9936	1057.368	1071.134	1072.722
	2.072	2.104	2.290	2.601	2.688	2.697	2.699
100.0	573.8491	588.7405	686.1675	924.4362	1028.728	1043.027	1044.676
	2.058	2.092	2.288	2.613	2.703	2.714	2.715
200.0	478.4866	494.9637	601.7785	859.1438	971.119	986.461	988.2311
	2.030	2.068	2.283	2.637	2.735	2.746	2.747
500.0	191.0209	212.6196	348.6686	661.8608	795.724	814.0344	816.146
	1.944	1.993	2.269	2.709	2.828	2.842	2.844
1000.0	−293.2226	−261.745	−72.8828	328.4788	495.2930	518.063	520.6890
	1.798	1.863	2.244	2.829	2.980	2.997	2.999
2000.0	−1283.257	−1226.979	−915.193	−354.424	−132.445	−102.197	−98.708
	1.532	1.614	2.191	3.062	3.260	3.282	3.284

where

$$A_{22} = \int_0^1 g_m(D^2 - k^2)g_i \, dZ, \quad B_{11} = Df_m(1)g_i(1),$$

$$C_{12} = \int_0^1 f_m D g_i \, dZ, \quad D_{12} = \int_0^1 f_m(1-Z)g_i \, dZ.$$

A_{11}, A_{12}, A_{21} and B_{12}, which appear in equations (35)–(37), are the same as those in the case of the a.c. electric field.

RESULTS AND DISCUSSION

Neutral curves were obtained by changing the wave number. The minimum values of the neutral curves are the critical electric Rayleigh number, the critical Rayleigh number and the critical Marangoni number. The number of trial functions was increased until the effective digits of those critical values reached at least six. Usually, the convergent solution was obtained with seven or eight trial functions.

Table 3. Critical Marangoni number and critical wave number under an a.c. electric field

	Bi					
Ea	0.0	0.1	1.0	10.0	100.0	1000.0
(a) $\Phi = 0$ at free surface						
0.0	$Ma_c = 79.60669$	83.42673	116.1271	413.4398	3303.83	32170.1
	$k_c = 1.993$	2.028	2.246	2.743	2.975	3.010
1.0	79.50997	83.32787	116.0097	413.1540	3301.92	32152.
	1.993	2.028	2.246	2.742	2.975	3.010
10.0	78.63864	82.43722	114.9524	410.5771	3284.68	31988.
	1.993	2.028	2.246	2.740	2.971	3.005
20.0	77.66871	81.44574	113.7750	407.704	3265.45	31806.
	1.993	2.029	2.245	2.737	2.967	3.000
50.0	74.74752	78.45936	110.2263	399.030	3207.29	31254.
	1.995	2.029	2.244	2.729	2.954	2.986
100.0	69.84048	73.44186	104.2566	384.3776	3108.74	30318.
	1.997	2.031	2.243	2.176	2.933	2.966
200.0	59.87850	63.25212	92.10547	354.3352	2905.57	23837.
	2.005	2.038	2.242	2.692	2.894	2.923
500.0	28.70804	31.34266	53.84702	258.095	2246.33	22106.
	2.048	2.078	2.259	2.642	2.803	2.826
1000.0	−28.64804	−27.43470	−17.12699	75.1131	969.52	9902.5
	2.212	2.233	2.364	2.629	2.732	2.746
2000.0	−186.5121	−188.832	−209.717	−418.6615	−2507.458	−23406.
	2.940	2.938	2.930	2.907	2.896	2.895
(b) $d\Phi/dZ$ at free surface						
0.0	$Ma_c = 79.60669$	83.42673	116.1271	413.4398	3303.83	32170.
	$k_c = 1.993$	2.028	2.246	2.743	2.975	3.010
1.0	79.49758	83.31688	116.0088	413.2136	3302.49	32158.
	1.993	2.028	2.246	2.743	2.975	3.010
10.0	78.51480	82.32754	114.9438	411.1755	3290.37	32045.
	1.990	2.026	2.245	2.742	2.975	3.01
20.0	77.42137	81.22680	113.7585	408.9059	3276.88	31919.
	1.988	2.023	2.243	2.742	2.974	3.01
50.0	74.13197	77.91527	110.1915	402.0647	3236.16	31539.
	1.980	2.017	2.239	2.740	2.972	3.007
100.0	68.91966	72.36546	104.2089	390.5536	3167.503	30899.
	1.969	2.006	2.233	2.739	2.970	3.004
200.0	57.48539	61.15351	92.1024	367.1133	3027.18	29589.
	1.947	1.986	2.223	2.737	2.967	3.000
500.0	23.25120	26.65806	54.64369	293.225	2580.305	25411.
	1.901	1.946	2.211	2.747	2.970	3.00
1000.0	−36.41874	−33.60360	−11.93038	156.027	1733.421	17469.
	1.891	1.948	2.265	2.814	3.018	3.046
2000.0	−172.233	−172.2572	−173.676	−205.956	−574.2	−4276.
	2.621	2.659	2.845	3.158	3.276	3.292

Onset of natural convection in an a.c. electric field

The critical a.c. electric Rayleigh number Ea_c and the critical wave number k_c for the onset of electric convection which have been obtained from the neutral curve are shown in Table 1, where two different boundary conditions at the free surface, that is, $\Phi = 0$ and $d\Phi/dZ = 0$, are considered. Both the critical a.c. electric Rayleigh number and the critical wave number increase as the Biot number increases. In other words, electric convection tends to be suppressed and the distance between each cell becomes shorter as the heat transfer rate becomes higher at the free surface.

The critical Rayleigh number Ra_c and the critical wave number k_c for the onset of buoyancy convection are shown in Tables 2(a) and (b). The critical Rayleigh number decreases as the a.c. electric Rayleigh number increases.

The critical Marangoni number Ma_c and the critical wave number k_c for the onset of Marangoni convection are shown in Tables 3(a) and (b). As in the case of buoyancy convection, the critical Marangoni number decreases as the a.c. electric Rayleigh number increases.

From the above-mentioned results, it is found that natural convection tends to occur in the a.c. electric field as the a.c. electric Rayleigh number increases. Following Nield's analysis [4, 5], the critical value for the onset of natural convection can be expressed as follows:

$$\frac{Ea}{Ea_c}+\frac{Ra}{Ra_c}+\frac{Ma}{Ma_c}=1 \tag{38}$$

where Ea_c, Ra_c and Ma_c are, respectively, the critical a.c. electric Rayleigh number in the absence of both buoyancy and Marangoni effects (see Table 1), the critical Rayleigh number in the absence of both electrical and Marangoni effects (see Table 2) and the critical Marangoni number in the absence of both electrical and buoyancy effects (see Table 3). Although the deviation of the linear relation (38) from the actual instability curves becomes larger as the Biot number increases, when Ea, Ra and Ma are positive, the deviation is within 4% according to the present calculation where the Biot number is changed from 0 up to 1000. Nonetheless, the system is stable as long as the relation $Ea/Ea_c+Ra/Ra_c+Ma/Ma_c<1$ is satisfied when Ea, Ra and Ma are positive.

In the case of 10 cs silicone oil ($\alpha \approx 2.86\times10^{-3}$ K^{-1}, $\varepsilon \approx 2.6\times10^{-11}$ F m^{-1}, $\kappa \approx 1.06\times10^{-7}$ m^2 s^{-1}, $\nu \approx 1.1\times10^{-5}$ m^2 s^{-1}, $\rho \approx 9.7\times10^2$ kg m^{-3}), if the depth of liquid layer is 10 mm, the temperature difference is 10 K and the initial electric field is 10^5 V m^{-1}, the a.c. electric Rayleigh number becomes 1850. This shows that it is quite easy to initiate the onset of electric convection in silicone oil.

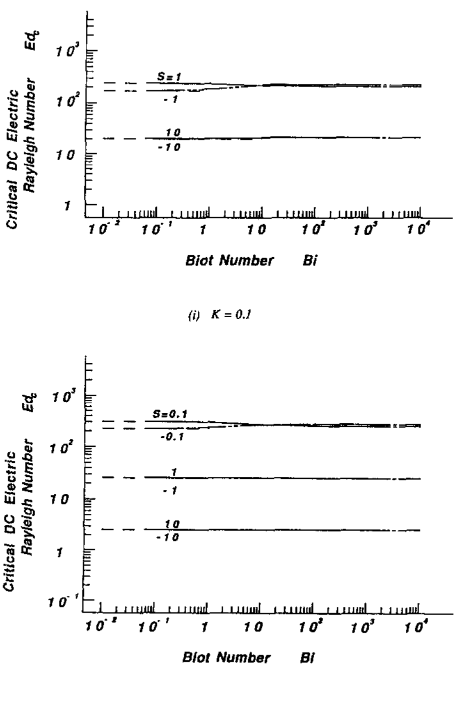

FIG. 2. Dependence of critical d.c. electric Rayleigh number on Biot number.

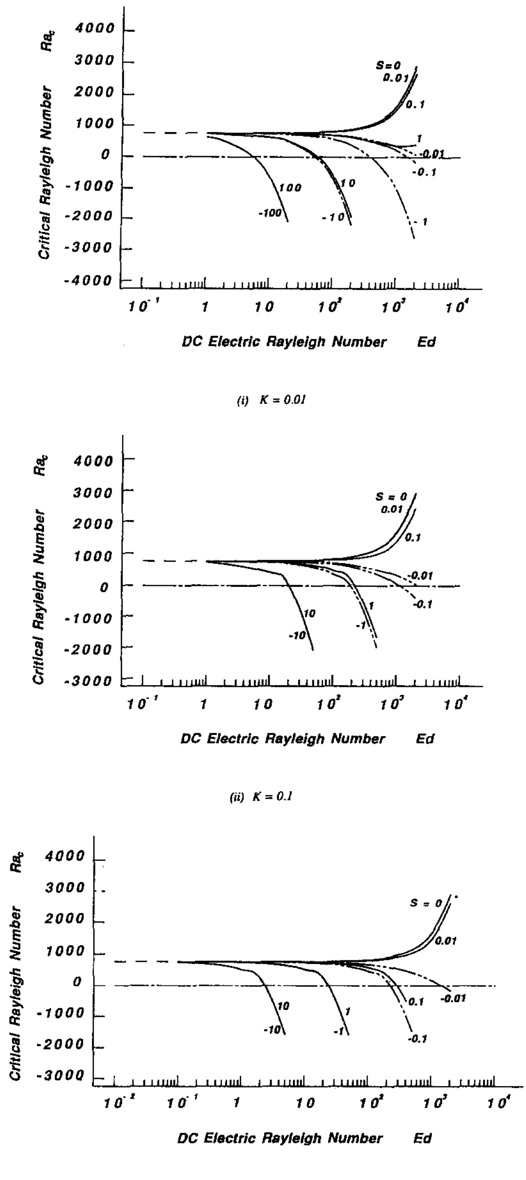

FIG. 3. Dependence of critical Rayleigh number on d.c. electric Rayleigh number.

Onset of natural convection in a d.c. electric field

The calculation was carried out for both positive and negative temperature coefficient s_1 (see equation (22)).

The dependence of the critical d.c. electric Rayleigh number Ed_c on the Biot number Bi is shown in Fig. 2, where S and K are taken as variables. As $|S|$ and K increase, Ed_c becomes smaller, that is, electric convection tends to occur.

The dependence of the critical Rayleigh number

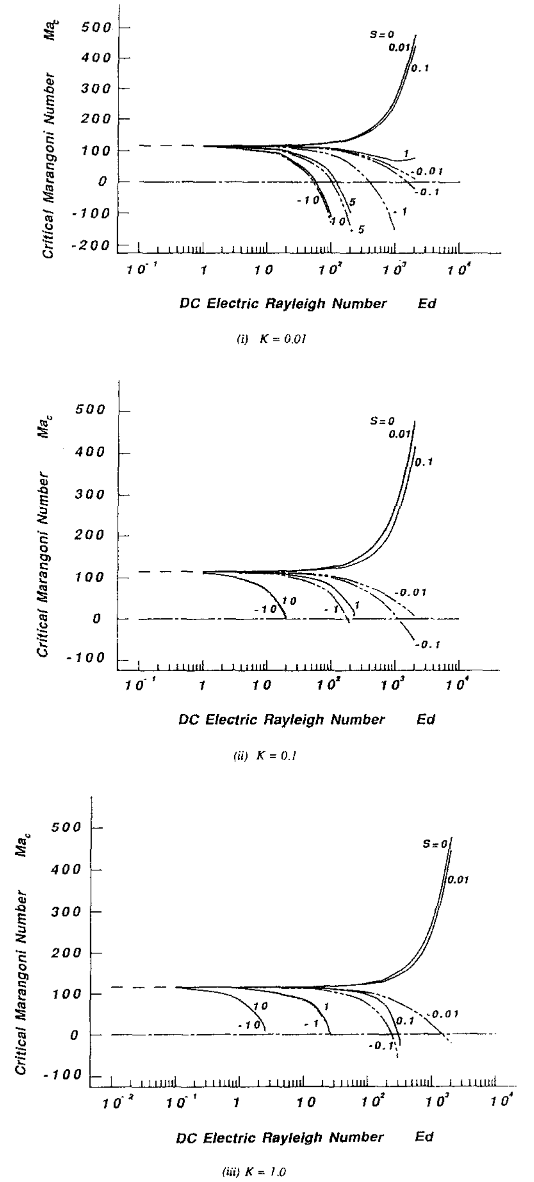

FIG. 4. Dependence of critical marangoni number on d.c. electric Rayleigh number.

Ra_c on the d.c. electric Rayleigh number Ed is shown in Fig. 3, where $Bi = 1$. When $|S|$ is very small, the critical Rayleigh number increases as the d.c. electric Rayleigh number increases, that is, buoyancy convection is suppressed under a strong d.c. electric field. However, Ra_c decreases as Ed increases when the nondimensional parameter K is large, even if $|S|$ is small.

The dependence of the critical Marangoni number Ma_c on the d.c. electric Rayleigh number Ed is shown in Fig. 4, where $Bi = 1$. As in the case of buoyancy convection, when $|S|$ is small, the critical Marangoni number increases as the d.c. electric Rayleigh number increases, and Ma_c decreases as Ed increases when the nondimensional parameters K and $|S|$ are large.

From the above-mentioned results, it is found that when the parameters K and $|S|$ are large, natural convection tends to occur as the d.c. electric Rayleigh number increases; on the contrary, convection is suppressed when K and $|S|$ are small.

In the case of 10 cs silicone oil ($s_1 \approx 3.8 \times 10^{-2}$ K^{-1}, $s_2 \approx 5.1 \times 10^{-4}$ K^2, $\sigma \approx 3.33 \times 10^{-14}$ S m^{-1}), if the depth of liquid layer is 10 mm, the temperature difference is 10 K and the initial electric field is 10^5 V m^{-1}, the nondimensional parameters K and S become 0.82 and 0.14, respectively, and the d.c. electric Rayleigh number becomes 8.6×10^5. This shows that it is also easy, as in the case of the a.c. electric field, to initiate the onset of electric convection in silicone oil.

CONCLUSIONS

The onset of natural convection in a horizontal liquid layer under a.c. and d.c. electric fields has been studied and the following results were obtained.

(1) In the case of the a.c. electric field, the a.c. electric Rayleigh number *Ea*, the Rayleigh number *Ra*, the Marangoni number *Ma* and the Biot number *Bi* become the governing parameters and natural convection tends to occur as the electric Rayleigh number increases.

(2) The d.c. electric Rayleigh number *Ed*, the Rayleigh number *Ra*, the Marangoni number *Ma*, the Biot number *Bi* and nondimensional parameters S and K are the governing parameters in the case of the d.c. electric field. When $|S|$ and K are large, natural convection tends to occur as the d.c. electric Rayleigh number increases. When K and $|S|$ are small, however, the convection is suppressed as the d.c. electric Rayleigh number increases.

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MISE EN PLACE DE LA CONVECTION NATURELLE SOUS UN CHAMP ELECTRIQUE

Résumé—On étudie théoriquement le problème de l'instabilité convective dans des champs électriques alternatifs (ac) et continus (dc). Les équations linéarisées de perturbation sont résolues par la méthode de Galerkin où le nombre de Rayleigh critique ac ou dc, le nombre critique de Rayleigh où le nombre critique de Marangoni sont exprimés en valeurs propres. Les paramètres adimensionnels de Biot *Bi*, de Rayleigh ac électrique *Ea*, de Rayleigh *Ra* et de Marangoni *Ma* déterminent la mise en place de la convection naturelle dans le cas du champ ac, tandis que *Bi*, *S*, *K*, *Ea*, *Ra* et *Ma* sont les paramètres dans le cas du champ électrique dc. On trouve qe le nombre critique de Rayleigh électrique, le nombre critique de Rayleigh et le nombre critique de Marangoni deviennent plus petits quand le champ électrique augmente, dans le cas du champ ac. Néanmoins, dans le cas du champ dc, le nombre critique de Rayleigh dc électrique, le nombre de Rayleigh critique et le nombre critique de Marangoni augmente quand le champ électrique croît si $|S|$ et K sont petits, alors que les mêmes grandeurs diminuent pour un champ électrique croissant si $|S|$ et K sont grands.

DAS EINSETZEN NATÜRLICHER KONVEKTION UNTER DER EINWIRKUNG EINES ELEKTRISCHEN FELDES

Zusammenfassung—Das Problem der konvektiven Instabilität in elektrischen Gleichstrom- und Wechselstromfeldern wird theoretisch untersucht. Linearisierte Störungsgleichungen werden mittels des Galerkin-Verfahrens gelöst, wobei die kritische Rayleigh-Zahl für Wechsel- oder Gleichstrom, die kritische Rayleigh-Zahl und die kritische Marangoni-Zahl als Eigenwerte beschrieben werden. Im Fall eines elektrischen Wechselfeldes wird das Einsetzen natürlicher Konvektion mit folgenden dimensionslosen Parametern beschrieben: Biot-Zahl *Bi*, Rayleigh-Zahl für elektrischen Wechelstrom *Ea*, Rayleigh-Zahl *Ra* und Marangoni-Zahl *Ma*. Im Fall eines gleichgerichteten elektrichen Feldes lauten die entsprechenden Parameter *Bi*, *F*, *K*, *Ea*, *Ra* und *Ma*. Es zeigt sich im Fall eines elektrischen Wechselfeldes, daß die kritische Rayleigh-Zahl für das Wechselfeld, die kritische Rayleigh-Zahl und die kritische Marangoni-Zahl mit zunehmender Feldstärke kleiner werden, während bei Gleichstromfeldern diese Kenzahlen mit wachsender Feldstärke steigen, sofern $|S|$ und K klein sind. Für große Werte für $|S|$ und K nehmen sie dagegen ab.

ВОЗНИКНОВЕНИЕ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ

Аннотация—Теоретически исследуется задача конвективной неустойчивости в электрических полях переменного и постоянного тока. Линеаризованные уравнения для возмущений решаются методом Галеркина, при этом значения критического числа Рэлея для электрического поля переменного или постоянного тока, а также критические числа Рэлея и Марангони выражаются через собственные значения. Возникновение естественой конвекции в случае электрического поля переменного тока определяется такими безразмерными параметрами, как число Био *Bi*, число Рэлея для электрического поля переменного тока *Ea*, а также числа Рэлея *Ra* и Маранкони *Ma*, в то время как параметры *Bi*, *S*, *K*, *Ea*, *Ra* и *Ma* являются определяющими в случае электрического поля постоянного тока. Найдено, что критическое число Рэлея для электрического поля переменного тока и критические числа Рэлея и Марангони уменьшаются с увеличением электрического поля. В случае же постоянного тока критическое число Рэлея для электрического поля постоянного тока, а также критические числа Рэлея и Марангони с увеличением электрического поля при малых значениях $|S|$ и K возрастают, а при больших значениях $|S|$ и K уменьшаются.